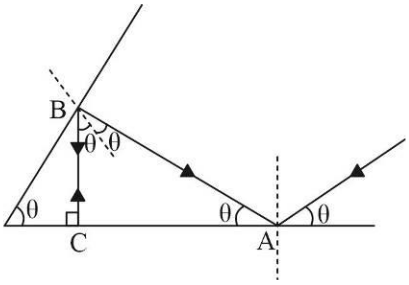
Solutions to JEE MAIN – 5 | JEE 2024

PHYSICS

SECTION-1

1.(B)



In ΔABC $\quad 90° + 3\theta = 180° \Rightarrow \theta = 30°$

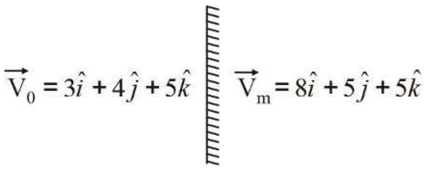
2.(B) $\dfrac{\lambda_{red}}{a} = \dfrac{3\lambda}{2a} \quad \therefore \quad \lambda = 4400\text{Å}$

3.(B) $V_{IZ} = 2V_{mz} - V_{oz} = 2 \times 8 - 5 = 11$

$V_{IX} = V_{ox} = 3$

$V_{IY} = V_{oy} = 4$

$\vec{V}_I = 3\hat{i} + 4\hat{j} + 11\hat{k}$

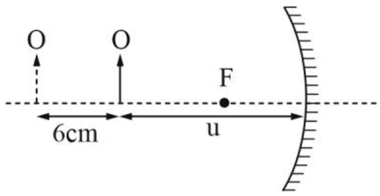


4.(A) Since the image is formed on screen it is real and inverted. Hence, $|u| > f$. As the object moves away from 'F' its image moves towards 'F' and size of image decreases.

$m = \dfrac{-v}{u} \Rightarrow -3 = \dfrac{-v}{(-u)} \Rightarrow v = -34u$

$\dfrac{1}{v} + \dfrac{1}{(-u)} = \dfrac{1}{(-f)} \Rightarrow \dfrac{1}{3u} + \dfrac{1}{u} = \dfrac{1}{f}$ . . . .(i)

Similarly $-2 = \dfrac{-v'}{(u+6)} \Rightarrow v' = -2(u+6)$



and $\dfrac{1}{v'} + \dfrac{1}{-(u+6)} = \dfrac{1}{(-f)}$

$\Rightarrow \quad \dfrac{1}{2(u+6)} + \dfrac{1}{(u+6)} = \dfrac{1}{f}$ . . . .(ii)

Solving (i) and (ii) we get

$u = 48cm \Rightarrow f = \dfrac{3u}{4} = 36cm$

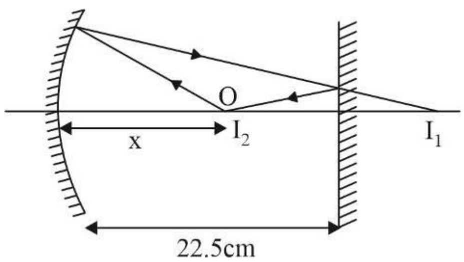
$v = 3u = 144cm$ and $v' = 2(u+6) = 108\,cm$

So, shift $= v - v' = 36\,cm$

5.(B) I_1 is the image formed by concave mirror

For reflection by concave mirror

$u = -x,\ v = -(45 - x),\ f = -10\,cm$



$$\frac{1}{-10} = \frac{1}{-(45-x)} + \frac{1}{-x}$$

$$\frac{1}{10} = \frac{x+45-x}{x(45-x)} \Rightarrow x^2 - 45x + 450 = 0 \Rightarrow x = 15cm, 30cm$$

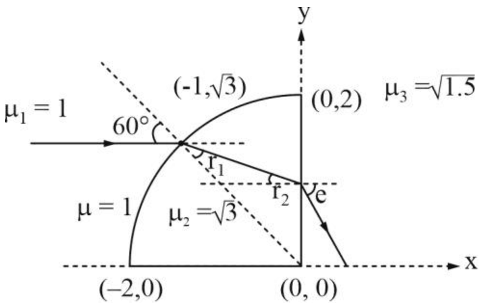
But x = 30 cm is not acceptable because x < 22.5 cm.

6.(C) For refraction on curved surface.

$$1\sin 60° = \sqrt{3}\sin r_1 \Rightarrow r_1 = 30°$$

And angle of deviation $\delta_1 = i - r_1$

$$\Rightarrow \quad \delta_1 = 60 - 30 = 30° \text{(clockwise)}$$



For refraction on plane surface.

$$r_2 = 60° - r_1 = 30°$$

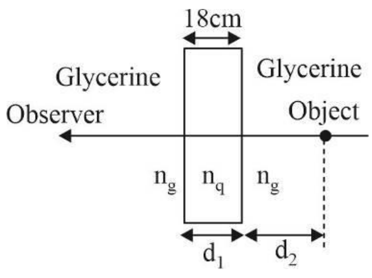
$$\sqrt{3}\sin r_2 = \sqrt{1.5}\sin e \Rightarrow e = 45°$$

and angle of deviation $\delta_2 = 15$ (Clockwise)

Net deviation $\delta_{net} = \delta_1 + \delta_2$

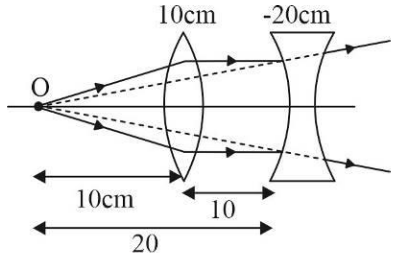
$\delta_{net} = 45°$ (clockwise)

7.(A) $n_{quartz} = 2\ ;\ n_{glycerine} = \frac{4}{3} \Rightarrow \frac{n_{quartz}}{n_{glycerine}} = \frac{2}{\frac{4}{3}} = \frac{3}{2} = \mu_{rel}$

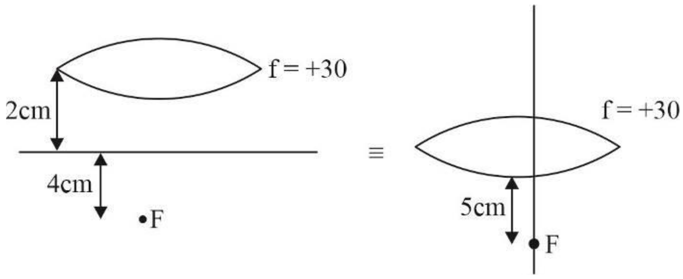


$$\text{Shift } = t\left(1 - \frac{1}{\mu_{rel}}\right) = 18\left(1 - \frac{1}{3/2}\right) = 6cm$$

8.(A)



9.(C)



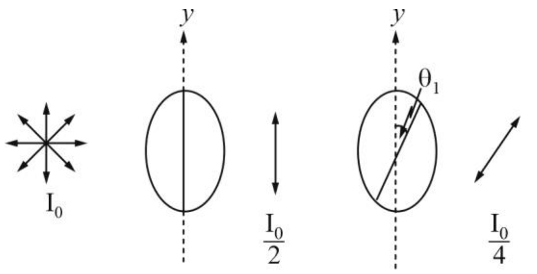
water $m = \frac{4}{3}$

$$\frac{1}{v} - \frac{1}{-5} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{6}{30}$$

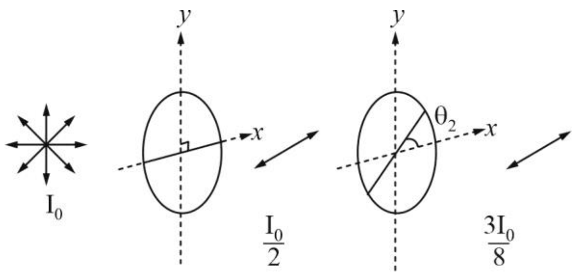
$$\frac{1}{v} = -\frac{5}{30}$$

$$v = -6\,cm$$

10.(B)



$$\frac{I_0}{4} = \frac{I_0}{2}\cos^2\theta_1 \quad \Rightarrow \quad \theta_1 = 45^\circ \text{ (with y – axis)}$$



$$\frac{3I_0}{8} = \frac{I_0}{2}\cos^2\theta_2 \quad \Rightarrow \quad \theta_2 = 30^\circ \text{ (with x-axis)}$$

Rotated angle $= (90 - \theta_2) - \theta_1 = 15^\circ$

11.(D) $\Delta x = (\mu_A - 1)t_A - (\mu_A - 1)t_B = \mu_A t_A - \mu_B t_B - t_A + t_B = t_B - t_A$

If $\Delta x > 0$, then fringe pattern will shift towards A

If $\Delta x < 0$, then fringe pattern will shift towards B

12.(B) For n[th] bright fringe

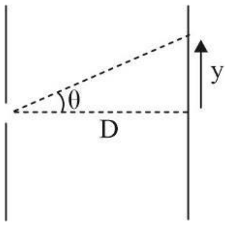
$$x = \frac{nD\lambda}{d}$$

$$D = \frac{xd}{n\lambda} = \frac{(18\times10^{-3})\times(5\times10^{-3})}{6(500\times10^{-9})} = 30 \text{ m}$$

13.(A) 2[nd] order minima for diffraction will be at [for n = 2]

$$\theta = \pm \frac{n\lambda}{a}$$

$$\theta = \pm \frac{2\lambda}{a}$$



$$\tan\theta = \frac{y}{D}$$

$$\theta \simeq \frac{y}{D}$$

$$y \simeq \theta D$$

$$y = \frac{2\lambda D}{a}$$

y is Position of 2[nd] order minima from central maxima
So, distance between two 2[nd] order maxima is,

$$2y = \frac{4\lambda D}{a}$$

= 2mm [after putting the values]

14.(B) $\delta = i + e - A$

$$30° = 15° + 60° - A$$

$$A = 45°$$

15.(B) $\delta_y = (\mu_y - 1)A$

$$= (1.62 - 1) \times 5°$$

$$= 0.62 \times 5 = 3.1°$$

$$\theta = \delta_B - \delta_R = \omega \times \delta_y$$

$$= 0.03 \times (1.62 - 1) \times 5$$

$$= 0.093°$$

16.(B) For maximum angular magnification the final image should be formed at the near point, So $v = -25cm$

$$\Rightarrow \quad f = 5cm \quad \Rightarrow \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \Rightarrow \quad \frac{1}{v} - \frac{1}{f} = \frac{1}{u}$$

$$\frac{1}{-25} - \frac{1}{5} = \frac{1}{u}$$

$$\frac{-(1+5)}{25} = \frac{1}{u}$$

$$u = \frac{-25}{6}cm = -4.16cm$$

17.(A) For myopic person the concave lens should focus the rays coming from infinity at a the far point of myopic person.

$\Rightarrow \; u = \infty, \; v = -3m$

$\Rightarrow \; \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{-1}{3} - \frac{1}{\infty} = \frac{1}{f}$

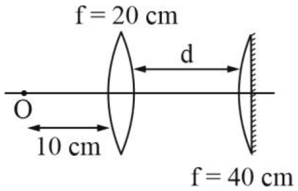
$f = -3m$

So, $P = \frac{1}{f} = \frac{1}{-3} = -0.33D$

18.(A) Number of fringes shifted $= \frac{(\mu - 1)t}{\lambda}$

$= \frac{(1.4-1)\times 1.2\times 10^{-5}}{480\times 10^{-9}} = \frac{0.4\times 1.2\times 10^{-5}}{480\times 10^{-9}} = 10$

19.(C) The silvered lens can be replaced by a mirror of focal length given as

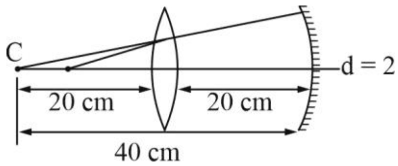


$\frac{1}{F_M} = \frac{1}{f_m} - \frac{2}{f_1}$

$\frac{1}{F_M} = 0 - \frac{2}{40} F_M = -20$

For lens $v = \frac{uf}{u+f}$

$v = \frac{-10\times 20}{-10+20} = -20$



So this position has to be centre of curvature of mirror in order for ray to retrace its path so $d = 40 - 20 = 20\,cm$

20.(C) $I_1 = I_0$

$I_2 = 0.75 I_0$ (Reflected light)

$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)^2$

$= \frac{(2+\sqrt{3})^4}{(4-3)^2} \approx 194$

SECTION-2

1.(1) $V_I = m^2 V_0$

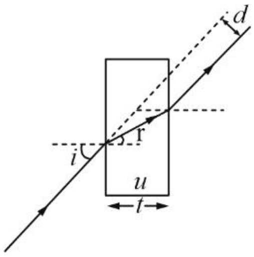
$= -\left(\frac{10}{30}\right)^2 \times 9 = 1 m/s$

2.(3) $d = \frac{t \sin(i-r)}{\cos r}$

$5\sqrt{3} = \frac{15 \sin(60-r)}{\cos r}$

$\frac{1}{\sqrt{3}} = \sin 60° - \cos 60° \left(\frac{\sin r}{\cos r}\right)$

$\tan r = \frac{1}{\sqrt{3}} \Rightarrow r = 30°$



Also, $1 \sin 60° = u \sin 30° \quad \Rightarrow \quad \mu = \sqrt{3}$

3.(9) For $e = 90°$, $r_2 = \theta c$

$\sin \theta c = \frac{1}{\mu_2} = \frac{1}{\sqrt{2}} \Rightarrow \theta c = 45°$

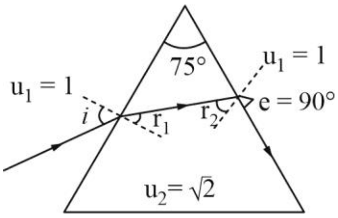
$r_1 = A - \theta c = 30°$

$u_1 \sin i = \mu_2 \sin r_1$

$\Rightarrow \quad 1 \sin i = \sqrt{2} \sin 30°$

$\Rightarrow \quad i = 45° = 5 \times 9$

$n = 9$



4.(20) For first refraction

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{3}{2 \times \infty} - \frac{1}{(-x)} = \frac{3/2 - 1}{+10}$

$\frac{1}{x} = \frac{1}{20} \Rightarrow x = 20 cm$

5.(90) For refraction through water surface,

$u = -12 cm$, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{4}{3v} + \frac{1}{12} = 0 \Rightarrow v = -16 cm$

Now, for lens,

$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$f_w = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_w} - 1\right)} \times f_a \qquad f_w = \frac{\frac{3}{2} - 1}{\left(\frac{3/2}{4/3} - 1\right)} \times 10 = 40\,cm$$

For refraction through the lens,

$u = -(16 + 44)\,cm = -60\,cm$

$f = +40\,cm$

$\therefore\ \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{40} - \frac{1}{60} = \frac{1}{120}$

$v = 120\,cm$

For refraction from water to glass slab,

$u = 120\,cm \quad \mu_2 = \frac{3}{2},\ \mu_1 = \frac{4}{3} \quad \frac{\frac{3}{2}}{v} - \frac{\frac{4}{3}}{120} = 0 \qquad (\because\ R = \infty)$

$v = 135\,cm$

Again for refraction from glass to air,

$u = 135\,cm \quad \mu_2 = 1,\ \ \mu_1 = \frac{3}{2}$

$\therefore\ \frac{1}{v} - \frac{\frac{3}{2}}{135} = 0 \qquad \therefore\ v = 90\,cm$

6.(5) Magnification by simple microscope when image is at near point is

$m = 1 + \frac{D}{f} = 1 + \frac{25}{6.25} = 5$

7.(400) For YDSE

$y = \frac{D\lambda}{d}$

$\lambda = \frac{yd}{D}$

$\lambda = \frac{(1\times10^{-3})(4\times10^{-3})}{10} = 4\times10^{-7}\,m$

$\lambda = 400\,nm$

8.(2) Let the intensity of individual waves be I, then

$I_0 = 4I \qquad \Rightarrow \qquad I = \frac{I_0}{4} \qquad \Rightarrow \qquad \Delta x \text{ at } P = d\sin\theta$

$\Delta x = \frac{d}{D} \times \frac{\beta}{4}$

$= \frac{d}{D} \times \frac{\lambda D}{4d} = \frac{\lambda}{4}$

$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \qquad [\Delta\phi = k\,\Delta x]$

$I' = I + 2\sqrt{I^2}\cos\frac{\pi}{2}$

$= 2I = \frac{I_0}{2}$

9.(20) $u_0 = -3.6cm$

$f_0 = +1.8cm$

For objective lens $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0}$

$= \frac{1}{1.8} - \frac{1}{3.6}$

$= \frac{2-1}{3.6}$

$\frac{1}{v_0} = \frac{1}{3.6}$

$v_0 = 3.6cm$

For normal adjustment the angular magnification of compound microscope is

$m = \frac{v_0}{u_0}\ \frac{D}{f_e}$

$m = \frac{3.6}{-1.8}\ \frac{25}{1.25} = -2\times 20 = -40$

10.(8) $P_1 = \frac{1}{f_1} = +5D$ and $P_{eq} = +2D$

$P_{eq} = P_1 + P_2$

$\Rightarrow \quad 2D = 5D + P_2$

$\Rightarrow \quad P_2 = -3D \Rightarrow \frac{1}{f_2} = -3$

For achromatic combination of lenses

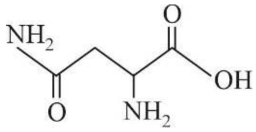
$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$

$\Rightarrow \quad \frac{\omega_1}{\omega_2} = \frac{-f_1}{f_2} = \frac{3}{5} \Rightarrow a+b=8$

CHEMISTRY

SECTION-1

1.(C) Asparagine has only one basic functional group in its chemical structure.



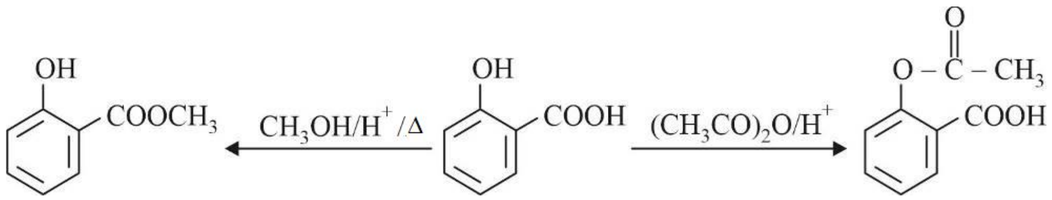
Others are basic amino acid with more than one basic functional group.

2.(B) $Cu(s) \rightarrow Cu(g) \rightarrow Cu^{+}_{(g)} \rightarrow Cu^{+2}_{(g)} \rightarrow Cu^{+2}_{(aq.)}$

Cu^{+2} is more stable that Cu^{+1} because released hydration energy is more in case of Cu^{+2} than Cu^{+1}.

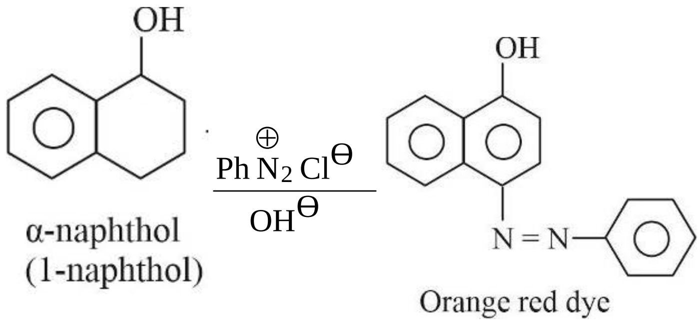
3.(C) Reduction of RNC will produce $R-NH-CH_3$.

4.(A)



5.(C) Ligand strength $\propto$ frequency of light absorbed.

6.(C)



7.(B) The carbylamine reaction is also known as Hoffmann isocyanide synthesis. It is the reaction of a primary amine, chloroform and a base to synthesize isocyanides.. The carbylamine reaction cannot be used to synthesize isocyanides from secondary or tertiary amines.

8.(C) Hinsberg reagent (Benzene sulphonyl chloride) is used to distinguish 1°, 2° and 3° amine. In the Hinsberg test, an amine is reacted with a benzene sulphonyl chloride. If a product forms, the amine is either a primary or secondary amine. Further if the product formed is soluble in base, then the amine has to be 1°.

9.(C) Vitamin A – Xerophthalmia

Vitamin B_{12} – pernicious anaemia

Vitamin C – Scurvy

Vitamin D – Rickets

10.(D) Fact based

11.(A) I. Glucose $\xrightarrow{HI}$ n – hexane

II. Glucose $\xrightarrow[H_2O]{Br_2}$ $\begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array}$

Gluconic acid

III. Glucose $\xrightarrow[\text{anhydride}]{\text{5 acetic}}$ Glucose pentacetate

IV. Glucose $\xrightarrow{HNO_3}$ $\begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \end{array}$

Saccharic acid

12.(B) Refer NCERT

13.(A) $R-\underset{\underset{O}{||}}{C}-Cl > R-\underset{\underset{O}{||}}{C}-O-\underset{\underset{O}{||}}{C}-R > R-\underset{\underset{O}{||}}{C}-OR > R-\underset{\underset{O}{||}}{C}-NH_2$

Reactivity towards Hydrolysis

14.(A)

15.(A) H_2O is weak field ligand and F^- and Cl^- are strong field ligands compared to $H_2O\cdot CuCl_4^2$ is bright green coloured.

16.(B)
- Nucleoside : A unit formed by the attachment of a base to 1 position of sugar is known as nucleoside.
- Nucleotides are joined together by phosphodiester linkage between 5' and 3' carbon atoms of the pentose sugar.
- Purine is bicycle heterocyclic aromatic organic compound that consists of two rings fused together (Guanine)
- Pyrimidine : Pyrimidine is monocycle aromatic heterocyclic compound (Thymine)

17.(D) Acidic strength $\propto \dfrac{1}{(+I)\text{ effect}}$

Acidic strength $\propto$ $(-I)$ effect

I. $\underset{(+I)}{CH_3}-COOH$

II. $\underset{(-I)}{F}-\underset{\underset{\underset{(-I)}{F}}{|}}{\overset{\overset{F(-I)}{|}}{C}}-COOH$

III. $\underset{(-I)}{Cl}-CH_2-COOH$

IV. $\underset{(-I)}{F}-CH_2-COOH$

V. $\underset{(-I)}{Br}-CH_2-COOH$

18.(C)

19.(C) Electron releasing group increases basic nature. Secondary amines are more basic than ammonia. Imides are not basic.

20.(C) $Cr^{+2}:[Ar], 3d^4, 4s^0\ n=4, \mu=\sqrt{4(4+2)}=\sqrt{24}=4.89\,BM$

$Mn^{+2}:[Ar], 3d^5, 4s^0\ n=5, \mu=\sqrt{5(5+2)}=\sqrt{35}=5.91\,BM$

$V^{+2}:[Ar], 3d^3, 4s^0\ n=3, \mu=\sqrt{3(3+2)}=\sqrt{15}=3.87\,BM$

$Ti^{+2}:[Ar], 3d^2, 4s^0\ n=2, \mu=\sqrt{2(2+2)}=\sqrt{8}=2.82\,BM$

SECTION-2

1.(4) $C_6H_5CONH_2+4KOH+Br_2 \longrightarrow C_6H_5NH_2+K_2CO_3+2KBr$

2.(5) $[Co(NH_3)_4Cl_2]Cl \Rightarrow$ Gives 1 mole AgCl

$[Ni(H_2O)_6]Cl_2 \Rightarrow$ Gives 2 moles AgCl

$[Pt(NH_3)_2Cl_2] \Rightarrow$ Gives No ppt of AgCl

$[Pd(NH_3)_4]Cl_2 \Rightarrow$ Gives 2 moles AgCl

Total number of moles of AgCl = 5 mole

3.(0) The number of bridging carbonyls in $Mn_2(CO)_{10}$ is 0. [Refer NCERT]

4.(6) The spin only magnetic moment of $[Mn(H_2O)_6]^{2+}$ complexes is 6 B.M.

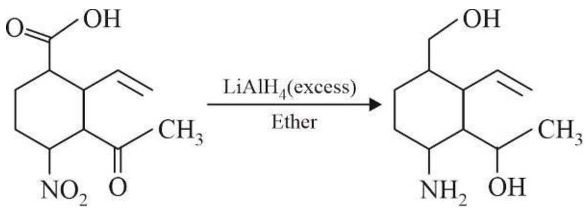
$[Mn(H_2O)_6]^{2+}$

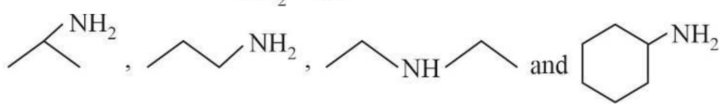
$Mn^{2+}=3d^5$

$\mu=\sqrt{5(5+2)}=5.91B.M.$

Nearest integer is 6.

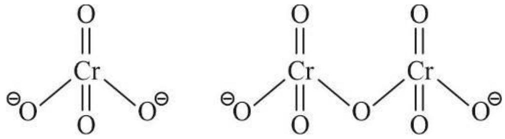
5.(1) $LiAlH_4$ reduces aldehydes, ketones and carboxylic acid to corresponding alcohols and nitro $(-NO_2)$ group to amino $(-NH_2)$ group. But $LiAlH_4$ cannot reduce carbon-carbon double bonds.

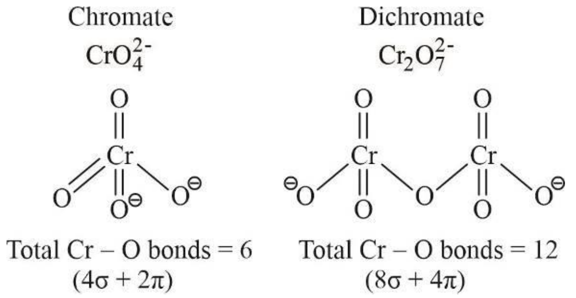


6.(5) Here $C_6H_5NH_2$, 

can reacts with it.

7.(18)

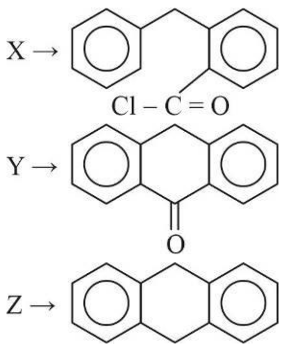




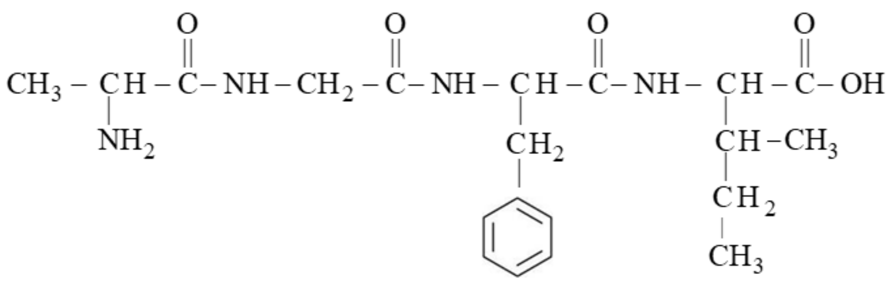
Total number of bonds between chromium and oxygen in both structures are 18.

8.(6) Statements 1, 2 and 3 are correct.

9.(3)



10.(10)



MATHEMATICS

SECTION-1

1.(D) $\dfrac{xdx+ydy}{ydx-xdy}=x^2+2y^2+\dfrac{y^4}{x^2}=\dfrac{\left(x^2+y^2\right)^2}{x^2}$

$\Rightarrow \quad \dfrac{xdx+ydy}{\left(x^2+y^2\right)^2}+\dfrac{xdy-ydx}{x^2}=0 \quad \Rightarrow \quad \dfrac{d\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2}+2d\left(\dfrac{y}{x}\right)=0$

$\Rightarrow \quad 2\dfrac{y}{x}-\dfrac{1}{\left(x^2+y^2\right)}=C$

2.(A) $I=\int \sin(100x+x).(\sin x)^{99}dx$

$=\int ((\sin(100x)\cos x+\cos 100x.\sin x)(\sin x)^{99})dx$

$\int \underbrace{\sin(100x)}_{I}\underbrace{\cos x.(\sin x)^{99}}_{II}dx+\int \cos(100x).(\sin x)^{100}dx$

$=\dfrac{\sin(100x)(\sin x)^{100}}{100}-\dfrac{100}{100}\int \cos(100x)(\sin x)^{100}dx+\int \cos(100x)(\sin x)^{100}dx$

$=\dfrac{\sin(100x)(\sin x)^{100}}{100}+C$

3.(B) Let $r=\vec{r}=\alpha\hat{i}+\beta j+\gamma k$ then $\vec{r}\times\left(\hat{i}+2j+k\right)=(\beta-2\gamma)\hat{i}+(\gamma-\alpha)j+(2\alpha-\beta)k.$

Thus $\beta-2\gamma=1, \gamma=\alpha.$ So $\vec{r}=\alpha\hat{i}+(1+2\alpha)j+\alpha k=j+\alpha\left(\hat{i}+2j+k\right).$

Also, it can be seen that $\hat{r}=j+\alpha\left(\hat{i}+2j+k\right)$ will satisfy $\vec{r}\times\left(\hat{i}+2j+k\right)=\hat{i}-k$ for any scalar α.

4.(A) $I_n=\int \tan^n x dx,\ (n>1)$

$I_4+I_6=\int(\tan^4 x+\tan^6 x)dx=\int \tan^4 x\sec^2 x dx$

Let $\tan x=t$

$\sec^2 x dx=dt$

$=\int t^4 dt=\left(\dfrac{t^5}{5}\right)+C=\left(\dfrac{1}{5}\right)\tan^5 x+C$

$a=\left(\dfrac{1}{5}\right), b=0$

5.(C) $\vec{a}=\dfrac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$

$\vec{b}=\dfrac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$

$|\vec{a}|=|\vec{b}|=1,\ \vec{a}\cdot\vec{b}=0$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|=|\vec{a}||\vec{b}|\sin 90^\circ=1$

$\left[2\vec{a}-\vec{b}\ \ \vec{a}\times\vec{b}\ \ \vec{a}+2\vec{b}\right]$

$=\left(\vec{a}\times\vec{b}\right)\cdot\left[(\vec{a}+2\vec{b})\times(2\vec{a}-\vec{b})\right]=\left(\vec{a}\times\vec{b}\right)\cdot 5(\vec{b}\times\vec{a})=-5\left(\vec{a}\times\vec{b}\right)^2=-5(1)=-5$

6.(D) $\vec{c} = \vec{b} \times \vec{a} \Rightarrow \vec{b} \cdot \vec{c} = 0$

$\Rightarrow \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \cdot \left(\hat{i} - \hat{j} - \hat{k}\right) = 0$

$b_1 - b_2 - b_3 = 0$

$\vec{a} \cdot \vec{b} = 3$

$\Rightarrow b_2 - b_3 = 3$

$b_1 = b_2 + b_3 = 3 + 2b_3$

Take $b_3 = -2$

$\vec{b} = (3 + 2b_3)\hat{i} + (3 + b_3)\hat{j} + b_3\hat{k}$

7.(A) $\left(3p^2 - pq + 2q^2\right)\left[\vec{u}\,\vec{v}\,\vec{w}\right] = 0$

But $\left[\vec{u}\,\vec{v}\,\vec{w}\right] \neq 0$

$3p^2 - pq + 2q^2 = 0$

$2p^2 + p^2 - pq + \left(\frac{q}{2}\right)^2 + \frac{7q^2}{4} = 0$

$\Rightarrow 2p^2 + \left(p - \frac{q}{2}\right)^2 + \frac{7}{4}q^2 = 0 \Rightarrow p = 0, q = 0, p = \frac{q}{2}$

8.(A) Vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$

are coplanar $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab$ $\quad \therefore$ a, c, b are in G.P.

9.(D) $\left(\vec{a} + 2\vec{b}\right) = t_1\vec{c}$ …(i)

and $\vec{b} + 3\vec{c} = t_2\vec{a}$ …(ii)

(i) – 2 × (ii) $\Rightarrow \vec{a}(1 + 2t_2) + \vec{c}(-t_1 - 6) = 0$

Since $\vec{a}$ and $\vec{c}$ are non-collinear

$\Rightarrow 1 + 2t_2 = 0 \Rightarrow t_2 = -1/2 \;\&\; t_1 = -6$

Putting the value of t_1 and t_2 in (i) and (ii), we get $\vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$

10.(B) $y = c_1 e^{c_2 x}$ …(i)

$y' = c_1 c_2 e^{c_2 x}$ …(ii)

$y'' = c_1 c_2^2 e^{c_2 x}$

$y'' = c_2 y'$ …(iii)

Now, (ii) / (i) $\frac{y'}{y} = c_2 \Rightarrow$ put in (iii)

$y'' = \frac{y'}{y} \cdot y' \Rightarrow y'' y = (y')^2$

11.(B) We have $\vec{a}\times\vec{b}=39\vec{k}=\vec{c}$

Also, $|\vec{a}|=\sqrt{34}, |\vec{b}|=\sqrt{45}, |\vec{c}|=39;$

$\therefore |\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39$

12.(B) $\int_{-4}^{3}|x^2-4|dx=\int_{-4}^{-2}|x^2-4|dx+\int_{-2}^{+2}|x^2-4|dx+\int_{2}^{3}|x^2-4|dx$

$=\int_{-4}^{-2}(x^2-4)\,dx+\int_{-2}^{2}(4-x^2)\,dx+\int_{2}^{3}(x^2-4)\,dx$

[as $|x^2-4|=4-x^2$ in $[-2, 2]$ and x^2-4 in other intervals]

$=\left|\frac{x^3}{3}-4x\right|_{-4}^{-2}+\left|4x-\frac{x^3}{3}\right|_{-2}^{2}+\left|\frac{x^3}{3}-4x\right|_{2}^{3}$

$=\left(-\frac{8}{3}+8\right)-\left(-\frac{64}{3}+16\right)+\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)+\left(\frac{27}{3}-12\right)-\left(\frac{8}{3}-8\right)=\frac{71}{3}$

13.(A) $I=\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx=2\int_0^{\pi}\frac{2x\sin x}{1+\cos^2 x}\cdot dx$ [Using : $\int_{-a}^{a}f(x)dx=\int_0^a[f(x)+f(-x)dx]$]

$\Rightarrow\ I=4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx\ \Rightarrow\ 2I=4\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$ [Using : $\int_0^a f(x)dx=\int_0^a f(a-x)\,dx$]

$\Rightarrow\ I=4\pi\int_0^{\pi/2}\frac{\sin x dx}{1+\cos^2 x}$ [Using : $\int_0^{2a}f(x)dx=\int_0^a[f(x)dx+\int_0^a f(2a-x)\,dx]$]

Put $\cos x=t\ \Rightarrow\ -\sin x dx=dt$

For $x=0, t=1$ and for $x=\frac{\pi}{2}, t=0$

$\Rightarrow\ I=4\pi\int_0^1\frac{dt}{1+t^2}=4\pi\tan^{-1}t\Big|_0^1=4\pi\frac{\pi}{4}=\pi^2$

14.(A) $\vec{a}+\vec{b}+\vec{c}=0\ \Rightarrow\ \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow\ \left(\vec{b}+\vec{c}\right)^2=\left(\vec{a}\right)^2\ \Rightarrow\ 5^2+3^2+2\vec{b}\cdot\vec{c}=7^2$

$\Rightarrow\ 2|\vec{b}||\vec{c}|\cos\theta=49-34=15$

$\Rightarrow\ 2\times5\times3\cos\theta=15$

$\Rightarrow\ \cos\theta=\frac{1}{2}\ \Rightarrow\ \theta=\frac{\pi}{3}=60^\circ$

15.(D) $\int_0^1\frac{(x^6-x^3)}{(2x^3+1)^3}dx=\frac{1}{2}\int_0^1\frac{2\left(1-\frac{1}{x^3}\right)}{\left(2x+\frac{1}{x^2}\right)^3}dx=-\frac{1}{36}$ (Put $2x+\frac{1}{x^2}=t$)

16.(B) $2\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{x}dx - \int_0^1 \frac{\tan^{-1}x}{x}dx$

Put $x = \sin\theta$ put $x = \tan\theta$

$2\int_0^{\frac{\pi}{4}} \frac{\theta\cos\theta}{\sin\theta}d\theta - \int_0^{\frac{\pi}{4}} \frac{\theta\sec^2\theta}{\tan\theta}d\theta = -\frac{\pi}{4}\ln 2 - 2\int_0^{\frac{\pi}{4}} \ln\sin\theta d\theta + \int_0^{\frac{\pi}{4}} \ln\tan\theta\, d\theta$ (using integration by parts)

$= -\int_0^{\frac{\pi}{4}} \ln\sin 2\theta\, d\theta = \frac{\pi}{4}\ln 2$

17.(A) We have, $\left[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right]$

$= \left(\vec{a}\times\vec{b}\right)\cdot\left\{\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right\}$

$= \left(\vec{a}\times\vec{b}\right)\cdot\left\{\left(\vec{m}\cdot\vec{a}\right)\vec{c} - \left(\vec{m}\cdot\vec{c}\right)\vec{a}\right\}$ (where $\vec{m} = \vec{b}\times\vec{c}$)

$= \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = 4^2 = 16$

18.(A) Given differential equation

$ydx + xy^2dx = xdy$

$\Rightarrow \frac{xdy - ydx}{y^2} = xdx \Rightarrow -d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right)$

Integrating we get $-\frac{x}{y} = \frac{x^2}{2} + C$ $\because$ It passes through $(1, -1)$

$\therefore\ 1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$ $\therefore\ x^2 + 1 + \frac{2x}{y} = 0 \Rightarrow y = \frac{-2x}{x^2+1}$ $\therefore\ f\left(-\frac{1}{2}\right) = \frac{4}{5}$

19.(A) $(x\log x)\frac{dy}{dx} + y = 2x\log x,\ (x \geq 1)$ $\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = 2$

I.F. $\Rightarrow e^{\int\frac{dx}{x\log x}} = e^{\log(\log x)} = \log x$

Solution is $y\log x = \int 2\log x dx$

$y\log x = 2x(\log x - 1) + C$ …(i)

Put $x = 1$ $y.0 = -2 + C$

$C = 2$ …(ii)

Put $x = e$ in (i)

$y\log e = 2e(\log e - 1) + C$

From (ii)

$y(e) = c = 2$

20.(B) $(2+\sin x)\dfrac{dy}{dx}+(y+1)\cos x=0$

$$\frac{d}{dx}(2+\sin x)(y+1)=0$$

$$(2+\sin x)(y+1)=c$$

$$x=0,\ y=1\Rightarrow c=4$$

$$y+1=\frac{4}{2+\sin x}$$

$$y\left(\frac{\pi}{2}\right)=\frac{4}{3}-1=\frac{1}{3}$$

SECTION-2

1.(1) Let $x^4=t$

$$4x^3dx=dt$$

$$\int_0^1\frac{1+t^{2010}}{(1+t)^{2012}}dt=\int_0^1\frac{1}{(1+t)^{2012}}dt+\int_0^1\frac{1}{t^2\left(1+\frac{1}{t}\right)^{2012}}dt=\left.\frac{(1+t)^{-2011}}{-2011}\right|_0^1+\left.\frac{\left(1+\frac{1}{t}\right)^{-2011}}{2011}\right|_0^1$$

$$\frac{-1}{2011}\left(\frac{1}{2^{2011}}-1\right)+\frac{1}{2011}\left(\frac{1}{2^{2011}}-0\right)=\frac{-1}{2011}\left(\frac{1}{2^{2011}}-1-\frac{1}{2^{2011}}\right)=\frac{1}{2011}=\frac{\lambda}{\mu}$$

2.(7) $\displaystyle\int_1^{\sqrt{3}}(x^{2x^2}x+2x^{2x^2}\cdot x\ln x)\,dx=\int_1^{\sqrt{3}}x^{2x^2}(x+2x\ln x)\,dx$

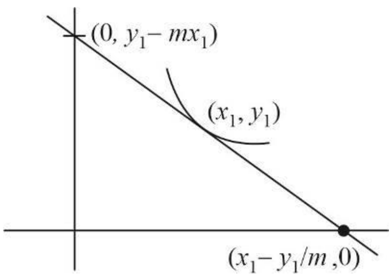
$$\int_1^{\sqrt{3}}(x^{x^2})^2(x+2x\ln x)\,dx\qquad\text{Let } x^{x^2}=t\Rightarrow x^2\ln x=\ln t\ ;\ (2x\ln x+x)\,dx=\frac{dt}{t}$$

$$\int_1^{(\sqrt{3})^3}t^2\cdot\frac{dt}{t}=\int_1^{(\sqrt{3})^3}t\,dt=\left.\frac{t^2}{2}\right|_1^{3^{3/2}}=\frac{3^3-1}{2}=13$$

3.(1) $x^a\cdot y=\lambda^a\ ;\ y=\dfrac{\lambda^a}{x^a}$

$$\frac{dy}{dx}=-a\lambda^a x^{-a-1}=-a\frac{x^a\cdot y}{x^{a+1}}$$

$$\Rightarrow\quad m=\frac{-ay_1}{x_1}$$



$$A=\frac{1}{2}|y_1-mx_1|\left|x_1-\frac{y_1}{m}\right|=\frac{1}{2}y_1x_1\frac{(1+a)^2}{a}$$

$$=\frac{1}{2}\lambda^a\cdot x_1^{1-a}(1+a)^2$$

For A to be constant $1-a=0$

4.(9) $I_{(6,8)}=\int_0^{\pi} x^6(\pi-x)^8dx=\left(-\frac{x^6(\pi-x)^9}{9}\right)_0^{\pi}+\int_0^{\pi}6x^5\frac{(\pi-x)^9}{9}dx$

$I_{(6,8)}=\frac{6}{9}I_{(5,9)}=\frac{6}{9}\times\frac{5}{10}\times\frac{4}{11}\times\frac{3}{12}\times\frac{2}{13}\times\frac{1}{14}\int_0^{\pi}(\pi-x)^{14}dx=\frac{6!\times 8!}{15!}\pi^{15}$

5.(3) Since $\hat{n}$ is perpendicular $\vec{u}$ and $\vec{v}$, $\hat{n}=\frac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$

$$\hat{n}=\frac{\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0\end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-\hat{k}$$

$|\vec{w}\cdot\hat{n}|=\left|(i+2j+3\hat{k})(-\hat{k})\right|=|-3|=3$

6.(1) $I=\int_2^4\frac{\log x^2}{\log x^2+\log(x-6)^2}dx$

$I=\int_2^4\frac{\log(6-x)^2}{\log x^2+\log(x-6)^2}dx$

(using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$)

$\therefore\ 2I=\int_2^4\frac{\log x^2+\log(6-x)^2}{\log x^2+\log(6-x)^2}dx=[x]_2^4=2\quad\therefore\ I=1$

7.(1) $\cos x dy=y(\sin x-y)dx\Rightarrow\frac{dy}{dx}=\frac{y\sin x-y^2}{\cos x}$

$\Rightarrow\ \frac{dy}{dx}=y\tan x-y^2\sec x\qquad\Rightarrow\ \frac{1}{y^2}\frac{dy}{dx}=\frac{1}{y}\tan x-\sec x$

$\Rightarrow\ \frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$

$\Rightarrow -\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}\tan x=\sec x$ ...(i)

Put $\frac{1}{y}=t$ in equation ...(i)

$\Rightarrow\ -\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$ ...(ii)

From equation (i) & (ii) we get

$\Rightarrow\ \frac{dt}{dx}+t\cdot\tan x=\sec x$

$\therefore\ I.F.=e^{\int\tan dx}=e^{\log|\sec x|}=\sec x$ $\quad\therefore$ solution of differential equation is

$$t \cdot \sec x = \int \sec x \cdot \sec x \cdot dx + c$$

$$\frac{1}{y}\sec x = \tan x + c$$

$$\sec x = y(\tan x + c)$$

$$y(0) = 1 \Rightarrow c = 1 \Rightarrow \sec x = y(\tan x + 1)$$

$$y = \frac{\sec x}{\tan x + 1} = \frac{1}{\sin x + \cos x}$$

$$y = \left(\frac{\pi}{2}\right) = 1$$

8.(7) $\frac{dy}{dx} = y + 3 > 0 \quad y(0) = 2, y(\log 2) = ?$

$$\int \frac{dy}{y+3} = \int dx$$

$$\log|y+3| = x + c$$

$$y(0) = 2$$

$$\log|2+3| = 0 + c \Rightarrow c = \log 5y$$

Put $(\log 2)\log|y+3| = \log 2 + \log 5$

$$\log|y+3| = \log 10$$

$$y + 3 = 10; y = 7$$

9.(3) $\int_0^{1.5} x[x^2]dx = \int_0^1 0dx + \int_1^{\sqrt{2}} xdx + \int_{\sqrt{2}}^{1.5} 2x\,dx$

$$\left[\frac{x^2}{2}\right]^{\sqrt{2}} + [x^2]_{\sqrt{2}}^{1.5} = \left(\frac{2}{2} - \frac{1}{2}\right) + (2.25 - 2)$$

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

10.(2) Put $x = \tan\theta$ then

$$I = \int_0^{\pi/4} \frac{8\log(1+\tan\theta)}{\sec^2\theta}\sec^2\theta d\theta \qquad \ldots(i)$$

$$I = 8\int_0^{\pi/4} \log(1+\tan\theta)d\theta = 8\left(\frac{\pi}{2}\log 2\right)$$

$$I = \pi \log 2$$